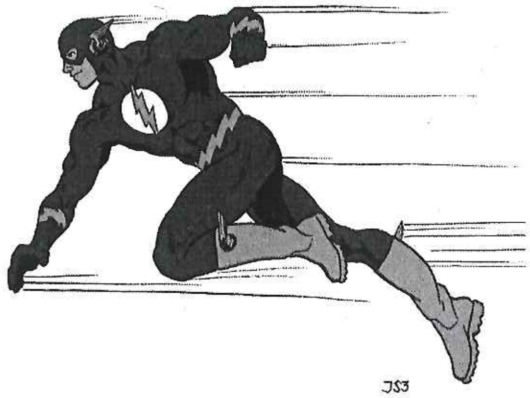
Physics 3310



Notes and Problem Sets
Work and Energy

Chapter 6

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KEY

STUDENT NAME

BRIENZA

TEACHER'S NAME

PERIOD

Lesson 1: Energy Basics

Energy = Motion in Mechanics

Motion = Speed

The more speed an object has the more energy it "carries."

Direction doesn't matter – Energy is a scalar, no $\hat{x}, \hat{y}, \hat{z}$ Yay!

Energy transferred to an Object = Object speeds up

Energy transferred away from an Object = Object slows down

How is this energy transferred?

Forces act on objects making them speed up or slow down thereby transferring energy to them or from them.

Work = Energy in Physics, it is "Energy" that a force produces when the force pushes or pulls on the mass

Need ① force that ② causes ③ displacement   ALWAYS need all 3 things!

Mathematical Definition of Work (Energy) – Dot Product

ex: A waiter holding up a tray is not doing work.

$$W \equiv \vec{F} \bullet \Delta\vec{x}$$

$$W = F\ (\Delta x)\cos\theta_{F,\Delta x}$$

W = the energy that a force transfers to or from an object as the object MOVES over a distance Δx

$\vec{F}$ $\vec{\Delta x}$

Units: 1 J = 1 N·m

1-D Example Problems:

1. A racecar is sitting on the track when a force of 1500 N to the East is exerted on it which moves it 10 m to the East. What is the angle between the force F and the displacement Δx? What is the work done by the force?

   $F = 1500N$ $\qquad W = F \Delta x \cos\theta$

   $\Delta x = 10m$ $\qquad = 1500N(10m)\cos 0$

   $\theta = 0°$ $\qquad \boxed{W = 15{,}000J}$

2. A racecar is moving down the track to the East when a force of 500 N to the West is exerted on it. The car continues to move 20 m while the force is exerted. What is the angle between the force F and the displacement Δx? What is the work done by the force?

   $F = 500N$ $\qquad W = F\Delta x \cos\theta$

   $\Delta x = 20m$ $\qquad = 500N(20m)\cos 180°$

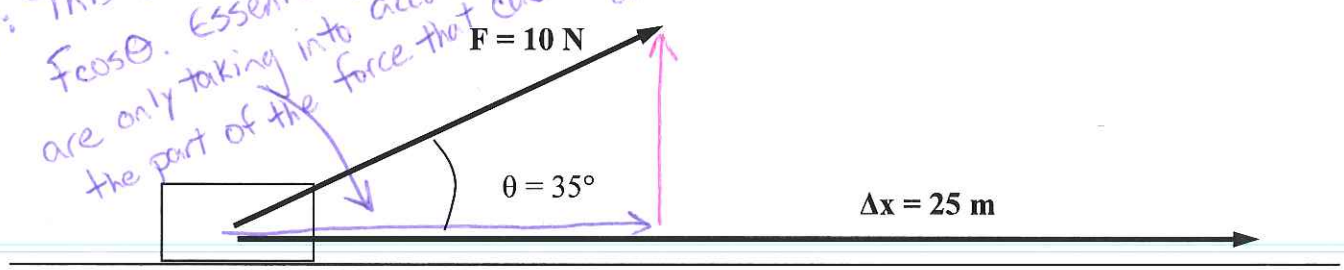
   $\theta = 180°$ $\qquad \boxed{W = -10{,}000J}$

2-D Dot Products

How to take the dot product of any 2 vectors.

1. Draw vectors from a common origin
2. Take angle between vectors. Note: MUST be between 0° and 180°
3. Multiply to get answer (no $\hat{x}, \hat{y}, \hat{z}$)

NOTE: This side is $F\cos\theta$. Essentially we are only taking into account the part of the force that causes the displacement!



Ex. Prob. 1: What is the work done by the above force?

$F = 10N$ $\qquad W = F\Delta x \cos\theta$

$\Delta x = 25m$ $\qquad = 10N(25m)\cos 35°$

$\theta = 35°$ $\qquad \boxed{W = 204.788J}$

Total Work (Net Work)

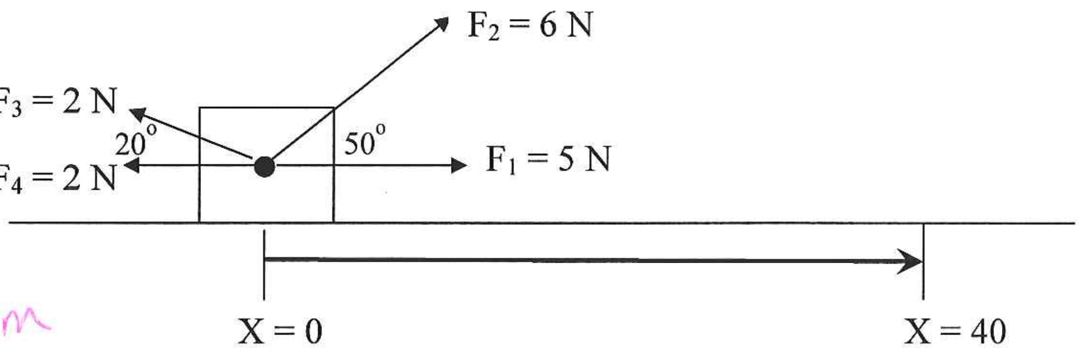
The total work done on a particle that moves from position x_1 to x_2 where $\Delta\vec{x} = \vec{x}_2 - \vec{x}_1$ is:

$$W = \left(\vec{F}_1 \bullet \Delta\vec{x}\right) + \left(\vec{F}_2 \bullet \Delta\vec{x}\right) + \left(\vec{F}_3 \bullet \Delta\vec{x}\right) + \ldots$$

$$W = \left(\vec{F}_1 \Delta\vec{x} \cos\theta\right) + \left(\vec{F}_2 \Delta\vec{x} \cos\theta\right) + \left(\vec{F}_3 \Delta\vec{x} \cos\theta\right) + \ldots$$

2-D Example Problem

Calculate the total work done on a mass m as it moves from position $x_1 = 0$ m to $x_2 = 40$ m



$\Delta x = 40m$

$F_1 = 5N$

$\theta_1 = 0°$

$W = 5N(40m)\cos 0°$

$W = 200J$

$F_2 = 6N$

$\theta_2 = 50°$

$W = 6N(40m)\cos 50°$

$W = 154.269J$

$F_3 = 2N$

$\theta = 160°$

$W = 2N(40N)\cos 160°$

$W = -75.175J$

$F_4 = 2N$

$\theta_4 = 180°$

$W = 2N(40m)\cos 180°$

$W = -80J$

TOTAL WORK:

$$\begin{array}{r} 200J \\ 154.269J \\ -75.175J \\ + \; -80J \\ \hline \boxed{199.09J} \end{array}$$

Lesson 1 Problems: Work

1. A 900-N crate rests on the floor. How much work is required to move it at constant speed (a) 6.0 m along the floor against a friction force of 180 N, and (b) 6.0 m vertically?

a. $F = 180N$

$\Delta x = 6m$

$\theta = 0°$

$W = 180N(6m)\cos\theta = \boxed{1080J}$

b. $F = 900N$

$\Delta x = 6m$

$\theta = 0°$

$W = 900N(6m)\cos 0 = \boxed{5400J}$

2. How high will a 0.325 kg rock go if thrown straight up by someone who does 115 J of work on it? Neglect air resistance.

$m = .325kg$

$W = 115J$

$F = F_g = mg = 3.185N$

$\theta = 0°$

$W = F\Delta x\cos\theta$

$115 = 3.185(\Delta x)\cos 0$

$\Delta x = \boxed{36.107m}$

3. A hammerhead with a mass of 2.0 kg is allowed to fall onto a nail from a height of 0.40 m. What is the maximum amount of work it could do on the nail? Why do people not just "let it fall" but add their own force to the hammer as it falls?

$m = 2kg$

$\Delta x = .4m$

$F = mg = 19.6N$

$\theta = 0°$

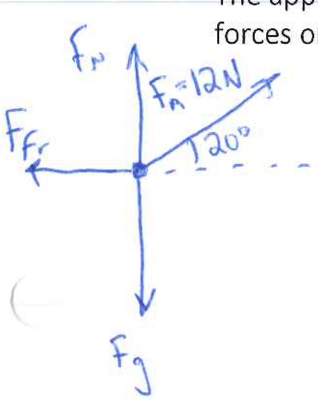
$W = F\Delta x\cos\theta$

$= 19.6(.4)\cos 0$

$\boxed{W = 7.84J}$

ADDING FORCE INCREASES THE AMOUNT OF WORK DONE TO THE NAIL

4. A grocery art with mass of 18 kg is pushed at constant speed along an aisle by a force F = 12 N. The applied force acts at a 20-degree angle to the horizontal. Find the work done by each of the forces on the cart if the aisle is 15 m long.



F_N AND F_g do not cause displacement so they do no work.

$W = F_A\Delta x\cos\theta$

$= 12N(15m)\cos 20° = \boxed{169.14J}$

$W = F_f\Delta x\cos\theta = (12\cos 20)(15m)\cos 180 = \boxed{-169.14J}$

5. Eight books, each 4.6 cm thick with mass 1.8 kg, lie flat on a table. How much work is required to stack them one on top of another?

$F = (1.8\,kg)(9.8\,m/s^2) = 17.64N$ $\therefore$ TOTAL $W = 28(17.64)(.046)$ $\boxed{W = 22.72024J}$

Book 1: $W = 0$

Book 2: $\Delta x = .046m$

$W = 17.64N(.046m) = .81144J$

Book 3: $W = 17.64N(2(.046m)) = 1.62288J$

Book 4: $W = 17.64N(3(.046m)) = 2.43432J$

Book 5: $W = 17.64N(4(.046)) = 3.24576J$

Book 6: $W = 17.64(5(.046)) = 4.0572J$

Book 7: $W = 17.64(6(.046)) = 4.86864J$

Book 8: $W = 17.64(7(.046)) = 5.68008J$

6. A 280-kg piano slides 4.3 m down a 30 degree incline and is kept from accelerating by a man who is pushing back on it parallel to the incline (Fig. 6-35). The effective coefficient of kinetic friction is .40. Calculate: (a) the force exerted by the man, (b) the work done by the man on the piano, (c) the work done by the friction force, (d) the work done by the force of gravity, and (e) the net work done on the piano.

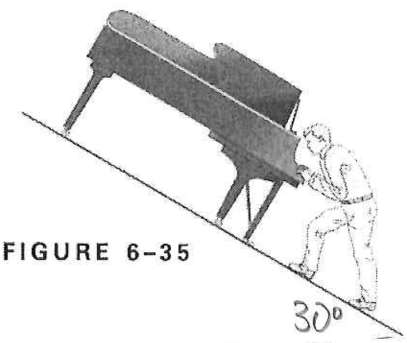
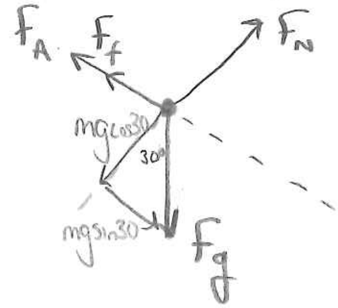


FIGURE 6-35

$m = 280kg$

$\Delta x = 4.3m$

$\mu = .4$



a. $\Sigma F = -F_A - F_f + F_g \sin\theta = 0$

$-F_A - \mu mg\cos 30 + mg\sin 30 = 0$

$F_A = -.4(280)(9.8)\cos 30 + 280(9.8)\sin 30$

$\boxed{F_A = 421.45N}$

b. $W = F\Delta x\cos\theta$ $\theta = 180°$

$= -421.45(4.3) \rightarrow \boxed{W = -1812.237N}$

c. $W = f\Delta x\cos\theta$

$= \mu mg\cos 30\Delta x\cos\theta = .4(280)(9.8)\cos 30(4.3)\cos 180 \rightarrow \boxed{W = -4087.36J}$

d. $F_g = mg = 2744N$

$\theta = 60°$

$W = F\Delta x\cos\theta$

$= 2744(4.3)\cos 60$

$\boxed{W = 5899.6J}$

e. $5899.6 - 4087.36 - 1812.237$

$\boxed{W = 0J}$ (NO ΣF SO W ALL "CANCELS")

Lesson 2: Work – Kinetic Energy Theorem

☐ For a constant net force, the acceleration is constant and we can use:

I. $\sum F_x = ma_x$ *(Newton's 2nd Law)*

II. $v^2 = v_i^2 + 2\,a_x\,\Delta x$ *(Const. Acceleration Equat.)*

Solve II. for Δx: $\Delta x = \dfrac{v_f^2 - v_i^2}{2a_x}$

Substitute Δx into $W_{net} \equiv F_x\,\Delta x$

$$= F_x\left[\frac{v_f^2 - v_i^2}{2a_x}\right]$$

$$= m\cancel{a_x}\left[\frac{v_f^2 - v_i^2}{2\cancel{a_x}}\right] \Rightarrow W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The quantity $\frac{1}{2}mv^2$ is called Kinetic Energy (KE)

$$\boxed{KE = \frac{1}{2}mv^2}$$

⟸ Energy due to motion
If an object has speed it has kinetic energy
Note: KE is a **scalar** quantity

$$\boxed{W_{done} = \Delta KE = \frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_i^{\,2}}$$

⟵ **Work – Kinetic Energy Theorem**

TO PROBLEM SOLVE USE

$$W_{done} = \vec{F} \bullet \Delta\vec{x} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Ex. Kinetic Energy is a Scalar Quantity Problem

A 3 kg ball is thrown at a wall with $\vec{v}_i = 5\ m/s\ \hat{x}$. The ball loses some energy when it strikes the wall. The ball rebounds with a velocity $\vec{v}_f = -4.2\ m/s\ \hat{x}$

a.) Calculate the change in KE of the particle.

$\Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$= \frac{1}{2}(3)(-4.2)^2 - \frac{1}{2}(3)(5)^2$

$\boxed{\Delta KE = -11.04 J}$

b.) If the final velocity of the ball is $\vec{v}_f = -5.0\ m/s\ \hat{x}$ what would be the net work done on the ball?

$\Delta KE = \frac{1}{2}(3)(-5)^2 - \frac{1}{2}(3)(5^2)$

$\Delta KE = \boxed{0 J = W}$

Lesson 2 Problems: Work – Kinetic Energy Theorem

7. At room temperature, an oxygen molecule, with mass of 5.31×10^{-26} kg, typically has a KE of about 6.21×10^{-21} J. How fast is it moving?

$m = 5.31\times10^{-26}\,kg$

$KE = 6.21\times10^{-21}\,J$

$v = ?$

$KE = \frac{1}{2}mv^2$

$6.21\times10^{-21} = \frac{1}{2}(5.31\times10^{-26})v^2$

$\boxed{v = 483.63\,m/s}$

8. (a) If the KE of an arrow is doubled, by what factor has its speed increased? (b) If its speed is doubled, by what factor does its KE increase?

a. $\underline{2}\,KE = \frac{1}{2}m(\underline{\sqrt{2}}\,v)^2 \rightarrow$ speed increased by $\sqrt{2}$

b. $\underline{4}\,KE = \frac{1}{2}m(\underline{2}\,v)^2 \rightarrow$ KE increased by 4

9. How much work is required to stop an electron ($m = 9.11 \times 10^{-31}$ kg) which is moving with a speed of 1.90×10^6 m/s?

$m = 9.11\times10^{-31}$

$v_i = 1.9\times10^{6}\,m/s$

$v_f = 0\,m/s$

$W = \Delta KE = \cancel{\frac{1}{2}mv_f^2}^{\,0} - \frac{1}{2}mv_i^2$

$W = -\frac{1}{2}(9.11\times10^{-31})(1.9\times10^{6})^2$

$\boxed{W = -1.644\times10^{-18}\,J}$

10. At an accident scene on a level road, investigators measure a car's skid mark to be 88m long. It was a rainy day and the coefficient of friction was estimated to be 0.42. Use these data to determine the speed of the car when the driver slammed on (and locked) the brakes. (why does the car's mass not matter?)

$\Delta x = 88m$

$\mu = .42$

$v_f = 0\,m/s$

$v_i = ?$

$W = F\Delta x\cos\theta = \cancel{\frac{1}{2}mv_f^2}^{\,0} - \frac{1}{2}mv_i^2$

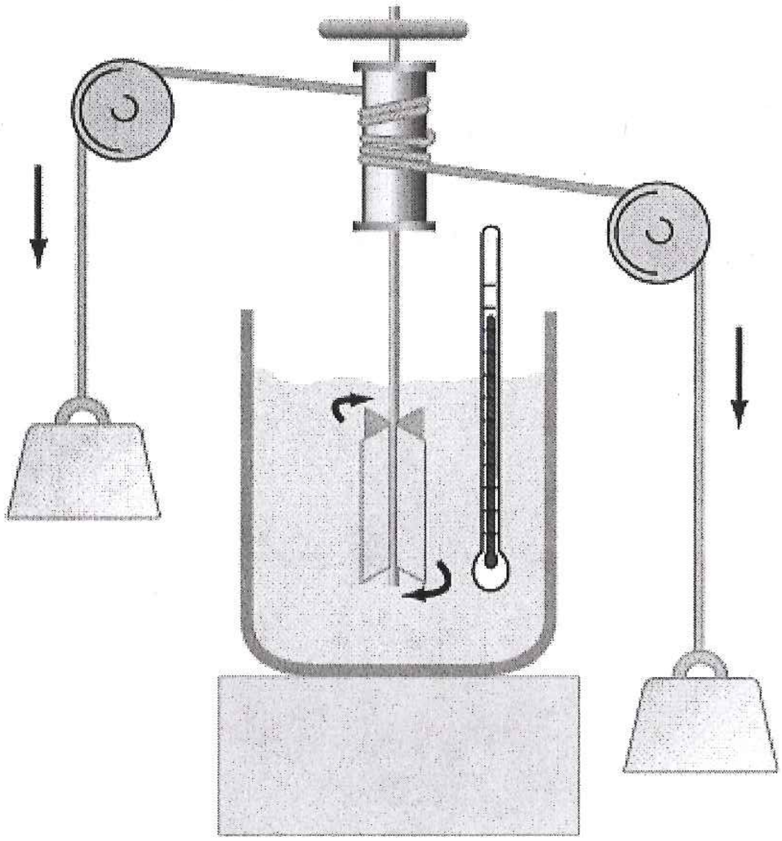
$\mu \cancel{m} g\,\Delta x\cos\theta = -\frac{1}{2}\cancel{m}v_i^2$

$+.42(9.8)(88)(\cos 180) = -\frac{1}{2}v_i^2$

$\boxed{v_i = 26.915\,m/s}$

11. One car has twice the mass of a second car, but only half as much kinetic energy. When both cars increase their speed by 5.0 m/s, they then have the same kinetic energy. What were the original speeds of the two cars?

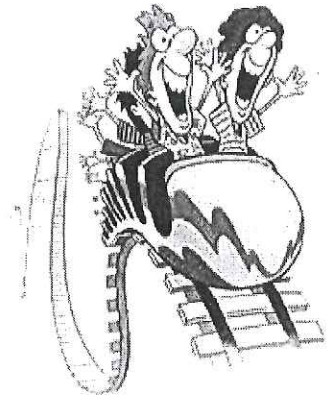
Lesson 3: Understanding Potential Energy



History of Science: Enrique Joule's Experiment

What happens as the masses drop?

Potential Energy → Kinetic Energy → Thermal Energy (Heat)



On the way up. Click, click, click. . .

$y_f = h = 10$ m ------------------------

As you lift a mass to height h, what is the work done by gravity?

$\Delta\vec{y}$

$\vec{F}_g$

$$W_g = \vec{F}_g \bullet \Delta\vec{y}$$
$$= mg(\Delta\vec{y})(\cos 180^o)$$
$$= -\,mg(y_f - y_i)$$

$W_g = -mgh$

$y_i = 0$ ------------------------

On the way down . . .

$y_f = h = 10$ m ------------------------

As the mass falls from height h, what is the work done by gravity?

$\Delta\vec{y}$ $\vec{F}_g$

$W = F\Delta y \cos 0^\circ$
$= mg\Delta y$
$W_g = mgh$

Gravity is a conservative force

$y_i = 0$ ------------------------

As you lift a mass against gravity, picture a roller coaster click-clicking up its very tall first hill. The motion, Δy, of the coaster is against the force of gravity, so gravity does negative work. The gravitational potential energy being ***stored*** is the following:

$$\therefore \qquad \Delta PE = -\,W_{\substack{\text{done by}\\ \text{gravity}}}$$

$$\Delta PE = +\,mg\,(y_f - y_i) = mgh$$

Then as the roller coaster reaches the top of the hill, all the potential energy is ***unloaded.*** The force of gravity does positive work (force and displacement vectors in line), and the roller coaster flies down the hill, converting all that *potential energy* into heart-in-your-throat *kinetic energy*.

The ***Mathematical Potential Energy function for Gravity*** as:

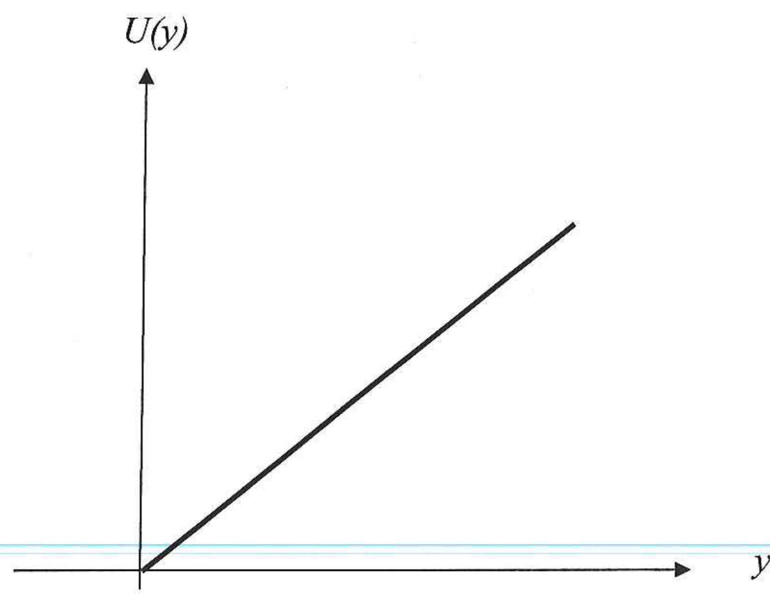
$$\boxed{U(y) \equiv mgy}$$

Potential Energy Functions

The value for the potential energy is given by the mathematical function that is used to calculate the work done by a conservative force

$$W_{gravity} = -\Delta(mgy)$$

$$\boxed{\therefore\; U(x)_{gravity} = mgy}$$



$$\boxed{PE_g = mgy}$$ -OR- $PE = mgh$

Conservation of Energy – No Friction Problem

Potential Energy is converted into Kinetic Energy and Kinetic Energy is converted into Potential Energy PERFECTLY <u>if</u> Friction is not present.

$$\boxed{(KE_i + PE_i) = (KE_f + PE_f)}$$

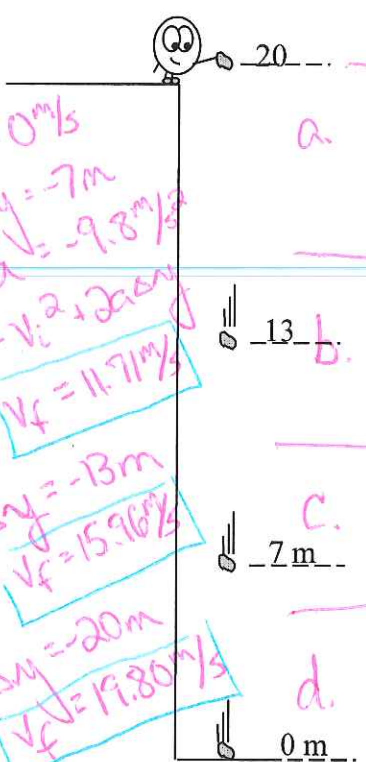
The **TOTAL ENERGY = KE + PE** at any point in time

In Physics

TOTAL ENERGY = ME (Mechanical Energy)

ME REMAINS CONSTANT IF NO FRICTION

EX. A 1kg mass is dropped straight down from a height of 20 m. For each of the heights below find KE, PE and ME. a.) y = 20 m b.) y = 13 m c.) y = 7 m d.) y = 0 m



$V_i = 0$ m/s
$\Delta y = -7$m
$a = -9.8$ m/s²
$V_f^2 = V_i^2 + 2a\Delta y$
$\boxed{V_f = 11.71 \text{ m/s}}$

$\Delta y = -13$m
$\boxed{V_f = 15.96 \text{ m/s}}$

$\Delta y = -20$m
$\boxed{V_f = 19.80 \text{ m/s}}$

	KE_i	PE_i	ME
a.	$\boxed{0J}$	mgh 1(9.8)20 $\boxed{196J}$	$\boxed{196J}$
b.	$\boxed{68.6J}$	mgh = 9.8(13) $\boxed{127.4J}$	$\boxed{196J}$
c.	$\boxed{127.4J}$	mgh = 9.8(7) $\boxed{68.6J}$	$\boxed{196J}$
d.	$\boxed{196J}$	$\boxed{0J}$	$\boxed{196J}$

e. lets find velocities at each location:

13m: $KE = 68.6 = \frac{1}{2}mv^2$
$68.6 = \frac{1}{2}(1)v^2$
$\boxed{V = 11.71 \text{ m/s}}$

7m: $KE = 127.4 = \frac{1}{2}mv^2$
$127.4 = \frac{1}{2}(1)v^2$
$\boxed{V = 15.96 \text{ m/s}}$

0m: $KE = 196 = \frac{1}{2}(1)v^2$
$\boxed{V \approx 19.799 \text{ m/s}}$

Lesson 3 Problems: Potential Energy; Conservation of Energy

12. Jane, looking for Tarzan, is running at top speed (5.6 m/s) and grabs a vine hanging vertically from a tall tree in the jungle. How high can she swing upward? Does the length of the vine affect your answer?

ALL KE converted to PE

$\therefore \frac{1}{2}\cancel{m}v^2 = \cancel{m}gh$

$\frac{1}{2}(5.6)^2 = 9.8h \rightarrow \boxed{h = 1.6m}$

13. A novice skier, starting from rest, slides down a frictionless 25 degree incline whose vertical height is 125 m. How fast is she going when she reaches the bottom?

ALL PE converted to KE

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$

$9.8(125) = \frac{1}{2}v^2 \rightarrow \boxed{v = 49.5 m/s}$

14. A roller coaster, shown in Fig. 6-38, is pulled up to point A where it and its screaming occupants are released from rest. Assuming no friction, calculate the speed at points B, C, and D.

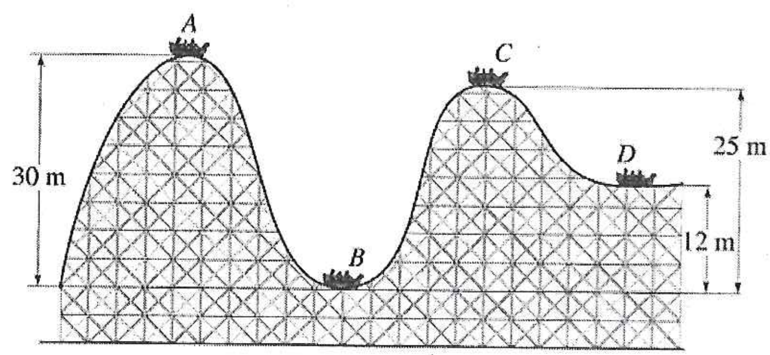


FIGURE 6–38

Point A: all PE

Point B: all KE

$\therefore PE_A = KE_B$

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$

$9.8(30) = \frac{1}{2}v^2$

$\boxed{v = 24.24 m/s}$

Point C: KE and PE

$PE_A = KE_C + PE_C$

$\cancel{m}gh_A = \frac{1}{2}\cancel{m}v^2 + \cancel{m}gh_C$

$9.8(30) = \frac{1}{2}v^2 + 9.8(25)$

$\boxed{v = 9.899 m/s}$

Point D: KE and PE

$PE_A = KE_D + PE_D$

$\cancel{m}gh_A = \frac{1}{2}\cancel{m}v^2 + \cancel{m}gh_D$

$9.8(30) = \frac{1}{2}v^2 + 9.8(12)$

$\boxed{v = 18.793 m/s}$

15. A projectile is fired at an upward angle of 45 degrees from the top of a 265 m cliff with a speed of 185 m/s. What will be its speed when it strikes the ground below? (Use conservation of energy.)

initial both KE and PE; final just KE

$$PE_i + KE_i = KE_f$$

$$\cancel{m}gh + \frac{1}{2}\cancel{m}v_i^2 = \frac{1}{2}\cancel{m}v_f^2$$

$$9.8(265) + \frac{1}{2}(185)^2 = \frac{1}{2}v_f^2$$

$$\boxed{v_f = 198.5\ m/s}$$

16. A small mass *m* slides without friction along looped apparatus shown in Fig. 6-39. If the object to remain on the track, even at the top of the circle (whose radius is *r*), from what minimum height must it be released?

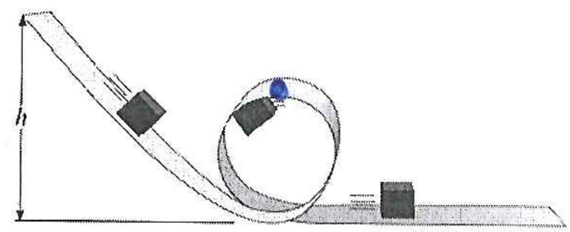


FIGURE 6–39

initial, just PE.

in order to stay on hoop, need enough PE at top of hoop (KE = 0 since it just makes the loop)

$$\cancel{mg}h = \cancel{mg}(2r)$$

$$\boxed{h = 2r}$$

Lesson 4: Conservation of Energy with Friction

If friction *is* present, it "robs" an object of some of its mechanical energy. The sum of kinetic and potential energies decreases as the frictional force does work on the object.

$$(KE_i + PE_i) \neq (KE_f + PE_f)$$

Specifically:

$$(KE_i + PE_i) > (KE_f + PE_f)$$

Example Problem:

A bicyclist (combined mass = 90 kg) starts down a hill of height 50 m with a velocity of 10 m/s and goes 55 meters up the next hill. How much work did friction do on the bicyclist? (How much PE and KE was lost?)

$$PE_i + KE_i + W_{nc} = PE_f + \cancel{KE_f} \rightarrow 0$$

since stops at the top of the hill

$$mgh + \tfrac{1}{2}mv^2 + W_{fr} = mgh$$

$$90(9.8)(50) + \tfrac{1}{2}(90)(10)^2 + W_{fr} = 90(9.8)(55)$$

$$44100 + 4500 + W_{fr} = 48510$$

$$\boxed{W_{fr} = -90J}$$

$$\boxed{KE_i + PE_i + W_{fr} = KE_f + PE_f}$$

$$W_{fr} = F_{fr}\Delta x\cos\theta$$

$$W_{fr} = \mu F_N \Delta x\cos\theta$$

$$\boxed{W_{fr} = \mu mg\Delta x\cos\theta}$$

FYI: Force of friction is always the opposite direction of the motion, so friction always does negative work, or takes energy rather than provides it.

Example Problem **Conservation of Energy – With Friction**

A ball bearing whose mass, *m,* is **0.0052 kg** is fired vertically downward from a height, *h*, of **18 m** with an initial speed v_o of **14m/s**. It buries itself in sand to a depth, *d*, of **0.21 m**. What average upward Frictional force, F_{fr}, does the sand exert on the ball as it comes to rest?

$m = .0052\,kg$

$h = 18\,m$

$V_i = 14\,m/s$

$\Delta x = .21\,m$

$F = ?$

$W = F\Delta x = \Delta KE$

$-\Delta PE = \Delta KE$ Since all the potential energy was converted to Kinetic energy

$PE_f - PE_i = mgh_f - mgh_i$

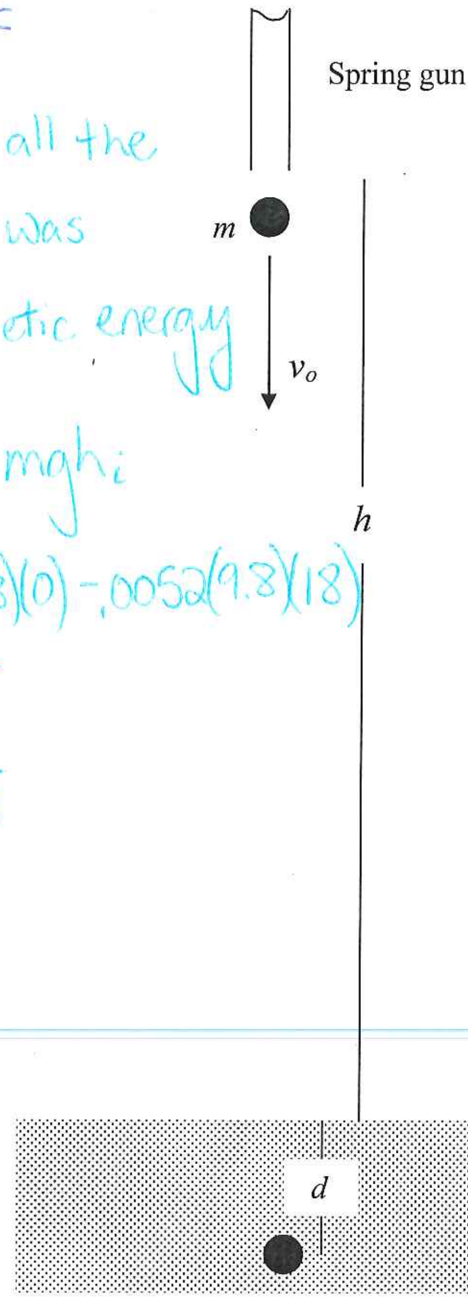
$= .0052(9.8)(0) - .0052(9.8)(18)$

$-\Delta PE = -.91728\,J$

$\Delta KE = .91728\,J$

$.91728 = F(.21)$

$\boxed{F = 4.368\,N}$



Lesson 4 Problems: Conservation of Energy with Friction

17. (II) A 17 kg child descends a slide 3.5 m high and reaches the bottom with a speed of 2.5 m/s. How much thermal energy due to friction was generated in this process?

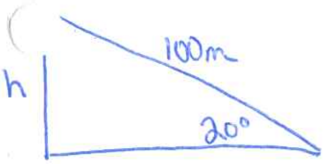
$$PE_i + \cancel{KE_i}^0 + W_{fr} = \cancel{PE_f}^0 + KE_f$$

$$mgh + W_{fr} = \frac{1}{2}mv^2$$

$$17(9.8)(3.5) + W_{fr} = \frac{1}{2}(17)(2.5)^2$$

$$\boxed{W_{fr} = -529.975\,J}$$

18. (II) A ski starts from rest and slides down a 20-degree incline 100 m long. (a) If the coefficient of friction is 0.090, what is the ski's speed at the base of the incline? (b) If the snow is level at the foot of the incline and has the same coefficient of friction, how far will the ski travel along the level? Use energy methods.



$\sin 20 = \frac{h}{100}$

$h = 34.202m$

a. $PE + W_{fr} = KE$

$$\cancel{m}gh + \mu\cancel{m}g\Delta x\cos\theta = \frac{1}{2}\cancel{m}v^2$$

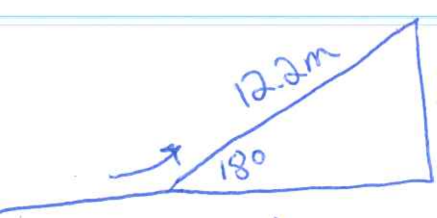
$$9.8(34.202) + .09(9.8)(100)\cos 180 = \frac{1}{2}v^2 \rightarrow \boxed{v = 22.225\,m/s}$$

b. $W_{fr} = \Delta KE$

$$\mu\cancel{m}g\Delta x\cos\theta = \frac{1}{2}\cancel{m}\cancel{v_f}^{2\,0} - \frac{1}{2}\cancel{m}v_i^2$$

$$+.09(9.8)\Delta x = +\frac{1}{2}(22.225)^2 \rightarrow \boxed{\Delta x = 280.02m}$$

19. (II) A skier traveling 12.0 m/s reaches the foot of a steady upward 18 degree incline and glides 12.2 m up along this slope before coming to a rest. What was the average coefficient of friction?



$\sin 18 = \frac{h}{12.2}$

$h = 3.77m$

$$KE_i + \cancel{PE_i}^0 + W_{fr} = \cancel{KE_f}^0 + PE_f$$

$$\frac{1}{2}\cancel{m}v^2 + \mu\cancel{m}g\Delta x\cos\theta = \cancel{m}gh$$

$$\frac{1}{2}(12)^2 + \mu(9.8)(12.2)\cos 180 = 9.8(3.77)$$

$$72 - 119.56\mu = 36.946$$

$$35.054 = 119.56\mu \rightarrow \boxed{\mu = .293}$$

Lesson 5: Power

Power = The rate at which energy is being used.

Average Power

$$\boxed{P = \frac{W}{\Delta t}}$$

Average time rate-of-change of work

$$W = \vec{F}\Delta\vec{x}\cos\theta$$

Substituting W into the equation above for P

$$P = \frac{\vec{F}\Delta\vec{x}\cos\theta}{\Delta t}$$

If F is constant, then $P = \vec{F}\frac{\Delta\vec{x}}{\Delta t}\cos\theta$

$$\boxed{P = Fv\cos\theta}$$

Power Units = Watts

$1W = 1J/s$

Lesson 5 Problem: Power

20. (I) How long will it take a 1750-W motor to lift a 285-kg piano to a sixth story window 16.0 m above?

$P = 1750W$

$m = 285kg$

$\Delta x = 16.0m$

$F_g = mg = 2793N$

$W = F\Delta x = 2793(16.0) = 44{,}688J$

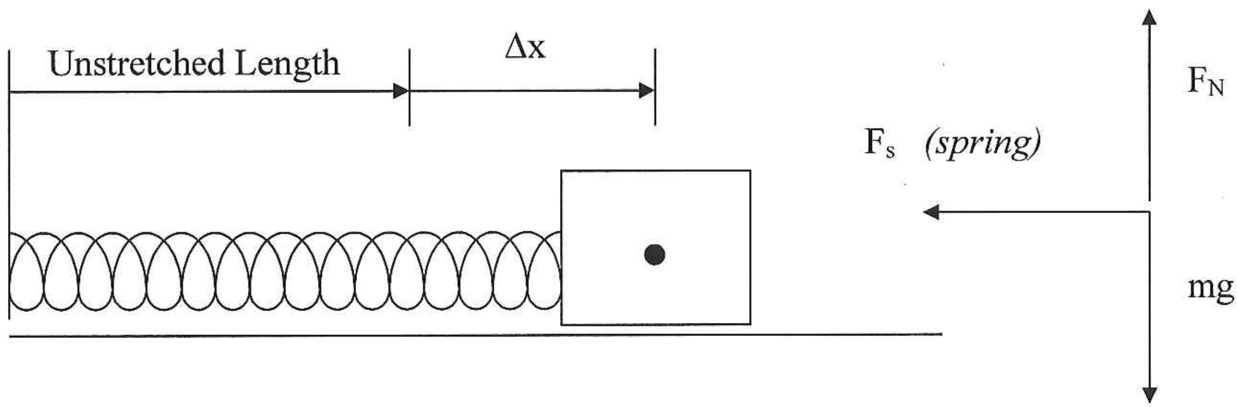
$P = W/t$

$1750 = \frac{44{,}688}{t}$

$\boxed{t = 25.536sec}$

Lesson 6 Springs

For Spring Forces – Hooke's Law



$$F_s(x) = -k \ \Delta x \qquad \textit{(SpringForce)}$$

☐ The minus sign means that the force exerted by the spring is always opposite to the stretching motion.

☐ The proportionality constant k is called the spring constant and different for different springs.

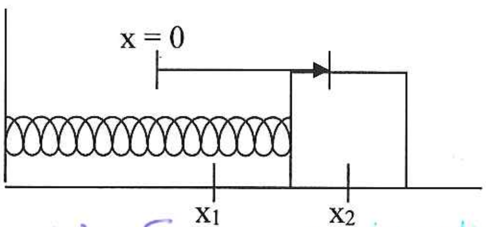
☐ Note: The spring force is always a restoring force. That is, it wants to restore the mass to the equilibrium position.

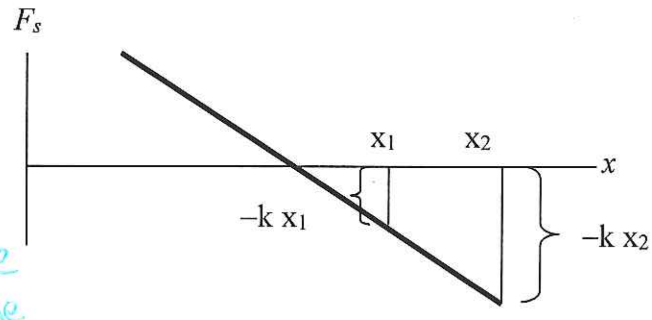
Spring Force Problem

A 2-kg block is connected to a spring with a force constant k=400 N/m as it oscillates on a frictionless, horizontal plane, as shown below. We choose the origin of the x-axis as shown so that the spring is unstretched and uncompressed at x = 0.

a.) Determine the work done by the net force on the block as it moves form $x_1 = 0.1$ m to $x_2 = 0.2$ m.

b.) If the speed of the block is 3 m/s when it is at x = 0.1 m , how fast is it moving as it passes the point $x_2 = 0.2$ m?





$W = F\,\Delta x$ ← Since the force varies, take the average force

a.) $W = \frac{1}{2}(F_{x_1} + F_{x_2})\,\Delta x$

$$W_{spring} = \frac{1}{2}(F_{x1} + F_{x2})\ \Delta x$$

Substitute $F = kx$

$$= \frac{1}{2}(-kx_1 - kx_2)\ (x_2 - x_1)$$

$$= \frac{1}{2}\left(-kx_1x_2 - kx_2^2 + kx_2x_1 + kx_1^2\right)$$

$$\boxed{W_{spring} = -\frac{1}{2}k\left(x_2^2 - x_1^2\right)} \neq -\frac{1}{2}k\,\Delta x^2$$

a. $W = -\frac{1}{2}(400)(.2^2 - .1^2) = -6\text{J}$

b. $W = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$-6 = \frac{1}{2}(2)v_f^2 - \frac{1}{2}(2)(3)^2$

$\boxed{v_f = 1.732\ \text{m/s}}$

$$W_{spring} = \int_{x_1}^{x_2} F_s(x)\ dx$$

$$= \int_{x_1}^{x_2} -kx\ dx$$

This is line integral with −kx as the integrand. Use the fundamental theorem of calculus to get:

$$W_{spring} = -\frac{kx^2}{2}\Bigg|_{x_1}^{x_2}$$

$$\boxed{W_{spring} = -\frac{1}{2}k(x_2^2 - x_1^2)}$$

Potential Energy Functions

The value for the potential energy is given by the mathematical function that is used to calculate the work done by a conservative force

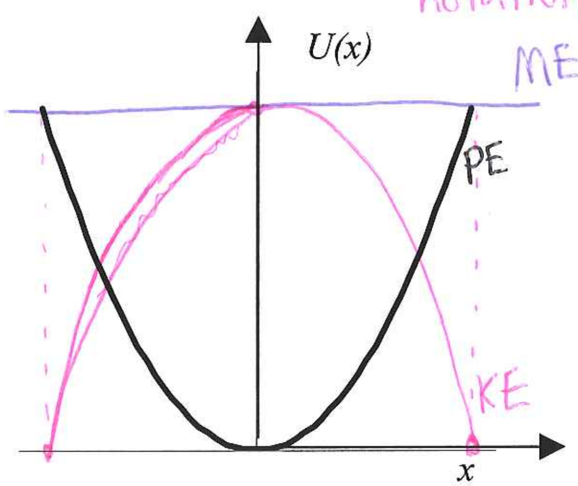
$W = -\frac{1}{2}k(x_f^2 - x_i^2)$

$$W_{spring} = -\Delta\left(\frac{1}{2}kx^2\right)$$

and $W = -\Delta PE$ so $\boxed{PE_s = \frac{1}{2}kx^2}$

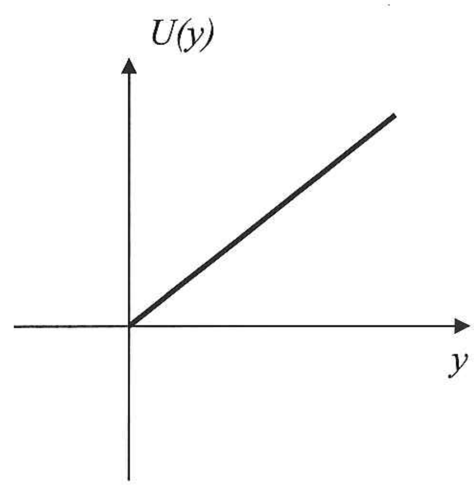
$$\boxed{\therefore\ U(x)_{spring} = \frac{1}{2}kx^2}$$

($U(x)$ is PE; just different notation)



$$W_{gravity} = -\Delta(mgy)$$

$$\boxed{\therefore\ U(x)_{gravity} = mgy}$$

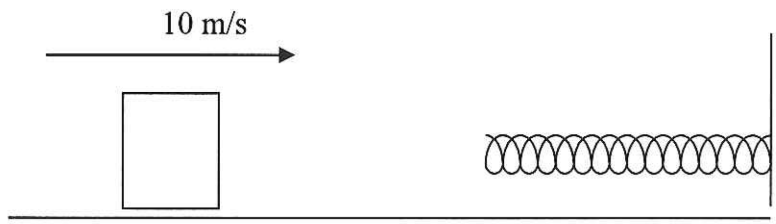


Work and Kinetic Energy Problem

The 2-kg block in the figure below is moving with a constant speed of 10 m/s on the horizontal, frictionless plane until it hits the end of the spring. The force constant of the spring is 200 N/m .

a.) How far is the spring compressed before the block comes to rest and reverses its motion?

b.) What is the speed of the block as the spring subsequently comes back to its original length?



$m = 2kg$

$v = 10 m/s$

$k = 200 N/m$

a. KE converted to PE_s

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$\frac{1}{2}(2)(10)^2 = \frac{1}{2}(200)x^2$

$\therefore \boxed{x = 1m}$

b. working backwards, PE_s converted to KE

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$\frac{1}{2}(200)(1)^2 = \frac{1}{2}(2)v^2$

$\boxed{v = 10 m/s}$

Lesson 6 Questions and Problems

23. You have two springs that are identical except that spring 1 is stiffer than spring 2 ($k_1 > k_2$). On which spring is more work done (a) if they are stretched using the same force, (b) if they are stretched the same distance?

$W = -\frac{1}{2}k(x_f^2 - x_i^2) = F\Delta x \quad F = -kx$

a. MORE ON K_2 (SPRING 1 STRETCHES LESS SO MORE WORK ON SPRING 2) b. MORE ON K_1 (MORE FORCE TO STRETCH SPRING 1 SO MORE WORK)

24. Describe the energy transformations when a child hops around on a pogo stick.

SPRING POTENTIAL → KINETIC → GRAVITATION POTENTIAL → KINETIC → SPRING POTENTIAL, etc.

25. A spring has a spring constant, k, of 440 N/ m. How much must this spring be stretched to store 25 J of potential energy?

$k = 440\ N/m$

$PE = 25J$

$\Delta x = ?$

$PE = \frac{1}{2}k\Delta x^2$

$25 = \frac{1}{2}(440)\Delta x^2$

$\boxed{\Delta x = .337m}$

26. A 75 kg trampoline artist jumps vertically upward from the top of a platform with a speed of 5.0 m/s. (a) how fast is he going as he lands on the trampoline, 3.0 m below (Fig. 6-37)? (b) If the trampoline behaves like a spring of spring constant 5.2×10^4 N/m, how far does he depress it?

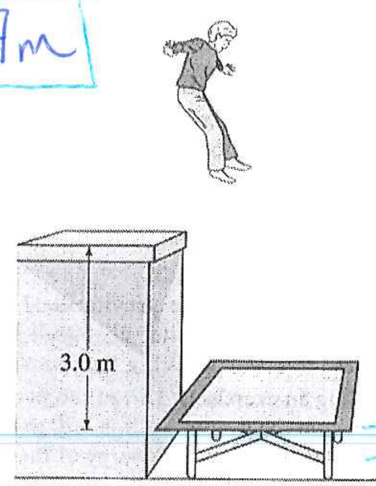


FIGURE 6-37

$y = h = x$

$y = 0$

a. $KE + PE = KE$

$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv^2$

$\frac{1}{2}(75)(5)^2 + 75(9.8)(3) = \frac{1}{2}(75)v^2 \rightarrow \boxed{v = 9.154 m/s}$

b. $KE + PE_g = PE_s$

$\frac{1}{2}mv^2 + mgh = \frac{1}{2}kx^2$ ← here h = how far spring stretchs, x

$\frac{1}{2}(75)(9.154)^2 + 75(9.8)x = \frac{1}{2}(5.2 \times 10^4)x^2$ ← QUAD FORM!

$-26,000x^2 + 735x + 3142.339 = 0$

$\boxed{x = .362m}$

27. A mass m is attached to the end of a spring (constant k) as shown if Fig. 6-40. The mass is given as initial displacement x_o from equilibrium and an initial speed v_o. Ignoring friction and the mass of the spring, use energy methods to find (a) its maximum speed and (b) its maximum stretch from equilibrium, in terms of the given quantities.

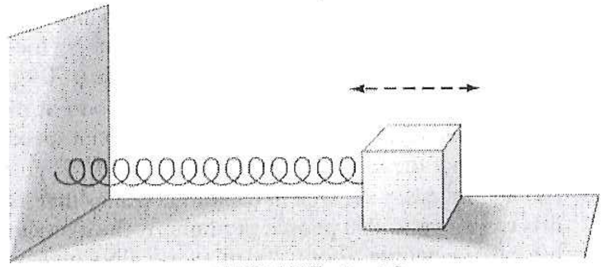


FIGURE 6–40

MASS m

$\Delta x = x_0$

$v_i = v_0$

a. MAX SPEED

$KE + PE_s = KE$

$\frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 = \frac{1}{2}mv^2$

$mv_0^2 + kx_0^2 = mv^2$

$$\boxed{v = \sqrt{\frac{mv_0^2 + kx_0^2}{m}}}$$

b. MAX STRETCH

$KE + PE_s = PE_s$

$\frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 = \frac{1}{2}kx^2$

$mv_0^2 + kx_0^2 = kx^2$

$$\boxed{x = \sqrt{\frac{mv_0^2 + kx_0^2}{k}}}$$

28. An elevator cable breaks when a 900 kg elevator is 30 m above a huge spring ($k = 4.0 \times 10^5$ N/m) at the bottom of the shaft. Calculate (a) the work done by gravity on the elevator before it hits the springs, (b) the speed of the elevator just before striking the spring, and (c) the amount the spring compresses (note that work is done by both the spring and gravity in this part).

$m = 900kg$

$h = 30m$

$k = 4 \times 10^5 N/m$

a. $W_g = +mgh\cos\theta$

$= +900(9.8)(30) \rightarrow \boxed{W = 264{,}600\,J}$

b. $PE_i = KE_f$

$mgh = \frac{1}{2}mv^2$

$9.8(30) = \frac{1}{2}v^2 \rightarrow \boxed{v = 24.25\,m/s}$

c. $KE_i + PE_G = PE_s$

$\boxed{y = 1.17m}$

$\frac{1}{2}mv^2 + mgy = \frac{1}{2}ky^2$

$\frac{1}{2}(900)(24.25)^2 + (900)(9.8)y - \frac{1}{2}(4\times10^5)y^2 = 0$

$264628.125 + 8820y - 200{,}000y^2$

QUAD FORM $\rightarrow$ $y = 1.17m$

27. A mass m is attached to the end of a spring (constant k) as shown if Fig. 6-40. The mass is given as initial displacement x_o from equilibrium and an initial speed v_o. Ignoring friction and the mass of the spring, use energy methods to find its maximum speed and (b) its maximum stretch from equilibrium, in terms of the given quantities.

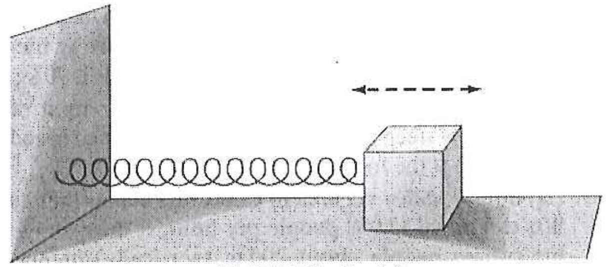


FIGURE 6–40

(a)

28. An elevator cable breaks when a 900 kg elevator is 30 m above a huge spring ($k = 4.0 \times 10^5$ N/m) at the bottom of the shaft. Calculate (a) the work done by gravity on the elevator before it hits the springs, (b) the speed of the elevator just before striking the spring, and (c) the amount the spring compresses (note that work is done by both the spring and gravity in this part).